

## **Thesis:** Logics of Resource and Justification

**Abstract:** In this report, I will overview the research results I have obtained in the fields of logics of justification and resources. The logics are a kind of explicit logics, that is, logics which make explicit some deep structure hidden in the standard logic systems like intuitionistic and classical logics. In Section 1, we report the new technique to realize a wide range of modal logics (epistemic logics) in justification logics. This method is modular and uniform for that range of modal logics. In Section 2, we propose a proof system for a basic logic known as the logic of provability in true arithmetic. The logic is basic and fundamental though a decent proof theory had not been explored. We offer the cut-elimination and some consequences of it: Craig interpolation theorem and definability theorem, and a proof-theoretic proof of the equivalence of the system and the well-known system GL with respect to Gödel sentences. In Section 3, we propose a generalization of the notion of constructive falsity proposed by Artemov 2019. Then, we completely characterize the closed sentences of Peano Arithmetic. Moreover, we locate Rosser sentences, constructive liar sentences and Reflection principles in the hierarchy of the generalized constructive falsity. In Section 4, we extend the meaning the modality of linear logic of the inexhaustibility of a proposition to mean additionally common resources sharable among agents in a setting of multi-agent system. Then, we give an extension of linear logic with such a modality using the tool of hypersequent calculus, and show the cut-elimination. We also prove decidability and realizability in terms of justification logic. In Section 5, we consider another extension of modality of linear logic where modal formulas  $s:A$  mean ‘ $s$  is available,  $A$  holds’, called ‘Linear Logic with explicit resources’. Then the usual modality is a special case of the empty resources. We show the cut-elimination and a type of realization theorem of linear logic to the pure fragment, that is, the subsystem of the extension without the original modality. In Section 6, we propose a bimodal substructural logic where one modality is the one in Linear Logic and the other is  $S4$  modality which does not satisfy structural rules. We prove the cut-elimination and the realization theorem via the justification counterpart with two sorts of terms. Also, we consider the subsystem which lacks structural rules at all and there occur no self-referentiality of constant specification of proofs.

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