Estimation of an Empirical FAVAR Model and DSGE Model for Evaluation of Government Expenditure Effects in Japan

Kohei Fukawa
City University of New York, Graduate Center
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Abstract

This paper studies the effects of government spending on the economy through estimation of an empirical Factor Augmented Vector Autoregression (FAVAR) model and a theoretical DSGE model. We first conducted FAVAR using 107 time series of Japan, and found that an increase in government investment and consumption leads to an increase in private consumption and real wages. We then set up a New Keynesian general equilibrium model with real and nominal rigidities, including both Edgeworth complementarity/substitutability between private and government consumption and productive public capital. Our model succeeds in private consumption and real wages increase in response to government expenditure shocks. We estimated key parameters of the model using Bayesian inference, and showed that private and government consumption are Edgeworth complements and that marginal productivity of public capital is productive in Japan.

1 Introduction

How does government expenditure affect the economy? This is a very old question. After the global financial crisis in the late 2000s, there has been renewed interest in the size of government spending multipliers. Large-scale-discretionary fiscal policies were undertaken, such as the America Recovery and Reinvestment Act of 2009 by the U.S. government, and huge successive economic policy actions in 2008 and early 2009 by the Japanese government. In Japan, a short-term macro model created by ESRI (an Error Correction model) was widely used for government multiplier arguments in Japan (Sakuma et al. (2011)). That model

\footnote{The act was comprised of approximately $500 billion in government spending and $290 billion in tax cuts, which means a total of 5.5% of GDP in the United States.}
(2011 version) declared a GDP multiplier of government investment equal to 1.07 in the first year and 1.14 for the second year.

To assess the effects of government spending, we first conducted an empirical study using a Factor Augmented Vector Autoregression (FAVAR) in this paper. As for empirical studies on fiscal policy effects, there have been several approaches and debates over the effects on private consumption and real wages.

The first approach is the so-called "government spending innovation approach," which uses a SVAR model and identifies a government spending shock using timing restrictions, which means government spending is not affected on impact by other shocks (i.e. Blanchard and Perotti (2002), Fatas and Mihov (2001), Gali et al. (2007), among others). This approach typically finds that an increase in public spending leads to a significant increase in private consumption and real wages.

The second approach is the "dummy variables identification approach," or the "narrative approach" (i.e. Ramey and Shapiro (1998), Burnside et al. (2004), Ramey (2011), among others). This approach tried to avoid the identification problem inherent in structural VAR analysis and instead looked for fiscal episodes that can be seen as exogenous with respect to the state of the economy. Ramey (2011) has argued that the large increases in military spending associated with the onset of the Korean war, the Vietnam war, the Carter-Reagan military buildup, and 9/11 can be seen as such exogenous events. During the episodes of large, exogenous increase in defense spending, output increases but private consumption and real wages fall, thus this approach gives smaller government multiplier.

The third approach uses a large dimensional dynamic factor model (i.e. Forni and Gambetti (2010)). Several criticisms of the VAR approach to policy shock identification focus on a small amount of information used by low dimensional VARs. To conserve degrees of freedom, standard VARs rarely employ more than 10 variables, even though this small number of variables is unlikely to span the information sets actually used by the government. Using low-dimensional VARs means that the measurement of policy innovation is likely to be contaminated. Forni and Gambetti (2010) studied the effects of government spending by using a structural dynamic factor model. They used principal components from more than 100 series, and used sign restriction for shock identification. They found government spending raises both consumption and investment with no evidence of crowding out.

This paper is of this third approach. We conducted FAVAR initiated by Bernanke, Boivin and Elizas (2005) using 107 time series; to make these series, I made reference to the Cabinet Office's "Monthly Economic Reports" and "Memorandum of Recent Economic Trends," two publications that, I believe,
the government actually uses for analysis of the Japanese economy. To identify a
government spending shock, we considered the timing of shocks; a government
spending shock is not affected on impact by other shocks (we used Choleski
decomposition).

We compared the results with preliminary VAR. We showed that the factors
included are Granger cause variable to most other variables included. The
impulse response of VAR and the FAVAR model showed both government in-
vestment shock and government consumption shock trigger an increase in pri-
vate consumption and output. Wages also increased after both shocks, although
there were sign changes in initial periods for the government consumption shock.
As far as we know, this is the first paper that applied FAVAR to analyze fiscal
policy shocks in Japan. We found that an increase in government investment
and consumption leads to an increase in private consumption and real wages,
which is in line with previous empirical studies.

This empirical crowd-in effect of private consumption and the increase in real
wages violate neoclassical macroeconomic theory, according to which govern-
ment spending decreases consumption. As explained by Baxter and King (1993),
the standard neoclassical model, one with infinitely-lived, forward-looking agents,
flexible prices, complete asset markets and a lump-sum taxation model, predicts
that increases in government spending create negative wealth effects by lowering
permanent household income. Although households typically increase labor
supply to prevent a large drop in consumption, this substitution effect is typ-
ically not strong enough to offset the wealth effect. In addition, this increase
in labor supply decreases real wages, but both of these results contradict with
empirical evidence.

Efforts aimed at reconciling this empirical evidence with theoretical modeling
have several approaches. The first direction is to adopt a non-separable prefer-
ence over consumption and leisure to strengthen the substitution effects after
the increase in real wages. Perotti and Monacelli (2008) employed the Green-
wood, Hercowitz, and Huffman preference in an otherwise standard neoclassical
model, and their calibrated model fit SVAR impulse in that consumption and
real wages increase in response to a government spending shock. This approach, however, seems
to be difficult to verify empirically.

Another direction is to embed the "deep habits" assumption of Ravn et al.
(2007) into imperfectly competitive product markets. This result is a model of
endogenous time-varying markups of price over marginal cost, and succeeded
in consumption and real wages increases in response to a government spending
shock, and there are some papers to estimate "deep habits" embedded model
such as Zubairy (2010).

Another direction is to adopt a credit-constrained agent (i.e. Gali et al.
(2007), Lopez-Salido and Rabanal (2006), Iwata (2011) among others), in which
a fraction of consumers is not allowed to optimize over the life-cycle. But a
widely recognized drawback of this approach is the excessive reliance on the
exogenous fraction of the credit-constrained agent to obtain a non-negative con-
sumption multiplier.

We adopt here an alternative approach that focuses on the presence of flow
of government consumption that is able to affect private consumption through modification of its marginal utility and productive government investment. Our motivation is based on the belief that government consumption and investment never deliver waste only; they enhance utility and production in the economy.

There have been attempts to explicitly consider Edgeworth complementarity / substitutability between private and government consumption into DSGE models, such as Bouakez and Rebei (2007), and Marattin and Marzo (2010). Bouakez and Rebei (2007) augmented a standard RBC model with complementarity between public and private consumption and habit formation to explain the crowd-in effects of private consumption, and estimated the parameters using U.S. data. They found a strong degree of complementarity between public and private consumption, and their estimated model succeeded in replicating the crowd-in effect of consumption but failed in replicating the real wage increase. Marattin and Marzo (2010) employed a New Keynesian equilibrium model with Edgeworth complementarity / substitutability between private consumption and government spending, capital adjustment cost, and distortional fiscal policy rules, and calibration on the Euro area. Their calibrated model was able to deliver positive private consumption multipliers while they gave no focus on the real wage movement.

There have also been attempts to consider productive government investment into general equilibrium models, such as Baxter and King (1993) and Leeper et al. (2009).

In addition, in Japan, many previous studies have estimated the production function with public capital, such as Mera (1973), Asako et al. (1994), Mitsui and Inoue (1995), Kawaguchi et al. (2005). For example, Mitsui and Inoue (1995) estimated production function using a macroeconomic time series and found the marginal product of public capital to be around 0.25. Kawaguchi et al. (2005) estimated the marginal product of public capital using exogenous variation in number of seats in the Diet by electoral reform in 1994 as an instrument variable, and found the elasticity to be around 0.2.

In this paper, we build on these contributions, and our key contributions of model sections are as follows. First, we extend Bouakez and Rebei (2007) model in the three respects: Our model is a New Keynesian setup, adding real rigidity (variable capital utilization) and productive public capital in production function. The model explicitly distinguishes government consumption and investment, and considered both Edgeworth complementarity/substitutability between private and government consumption and productive public capital.

\[ \text{Marginal Product of Public Capital} \approx 0.25 \]

\[ \text{Elasticity of Public Capital} \approx 0.2 \]

\[ \text{There have been also attempts to consider both productive government investment and Edgeworth complementarity/substitutability between private and government consumption into general equilibrium models, such as Mazraani (2010) and Iwata (2012).} \]

Mazraani (2010) augmented a standard RBC model with a distinction to government consumption and investment, using CES specifications over composite consumption and capital. She estimated the parameters using U.S. data, and found that both government consumption and capital complement private consumption and capital, and this results in the crowd-in to private consumption and private investment in response to government expenditure, although this model could not replicate the increase in real wages in initial periods.

Iwata (2012) established a medium-scale small open economy DSGE model augmented...
This model succeeds in private consumption and real wages increase in response to government expenditure shock.

Second, we estimated key parameters, i.e. elasticity of substitution between private consumption and government consumption or productive public capital share in production function, jointly with other structural parameters in a general equilibrium model using Japanese data. As for Edgeworth complementarity/substitutability between private and government consumption, we found that private and government consumption are Edgeworth complements, as Boaukez and Rebei (2007) found for U.S. data and Okubo (2003) found for Japanese data. As for marginal productivity of public capital in production function, the parameter was 0.30, which is larger than the estimate of 0.25 by Mitsui and Inoue (1995) and 0.20 by Kawaguchi et al. (2005). Also, from the estimated model, we showed the impact output multiplier to government consumption shock amounts to 1.81, while that to government investment shock is 0.75. In addition, we compared the impulse response of government spending shocks of estimated DSGE model with VAR and FAVAR models.

Third, we showed in the appendix that our model of new Keynesian setup with investment adjustment cost succeeds in more inertial movement in investment. Also, We showed that variable capital utilization helps dampen the large surge of rental rate of capital in response to government consumption or government investment shock, as pointed out by Christiano, Eichenbaum and Evans (2005).

The remainder of this paper is organized as follows. Section 2 discusses the empirical VAR and FAVAR models as a benchmark model. In Section 3, we set up the model. Section 4 shows the estimation result, and Section 5 concludes.

2 Empirical Evidence

The purpose of this section is to construct and estimate a benchmark VAR and FAVAR to illustrate the macroeconomic effects of a government spending shock.\footnote{To estimate VAR and FAVAR, we used Matlab codes and Eviews code provided by Carlo A. Favero in his HP.}

2.1 FAVAR framework:

I would like to first explain about Factor Augmented VAR.

\[
\begin{pmatrix}
Y_t \\
F_t
\end{pmatrix} =
\begin{bmatrix}
\Phi_{11}(L) & \Phi_{12}(L) \\
\Phi_{21}(L) & \Phi_{22}(L)
\end{bmatrix}
\begin{pmatrix}
Y_{t-1} \\
F_{t-1}
\end{pmatrix} +
\begin{pmatrix}
u^Z_t \\
u^F_t
\end{pmatrix}
\]  

with non-separability between private and public consumption, and productive public capital. He estimated the parameters using Japanese data. He found that government consumption complement private consumption and government investment is productive in Japan (marginal utility productivity of public capital is 0.046). His model, however, is a small open economy model, it is not suitable to see the movement of real wages.
$Y_t$ is the $(M \times 1)$ vector of observable variables and is assumed to have pervasive effects throughout economy, and $F_t$ is the $(k \times 1)$ vector of unobserved factors that captures additional economic information relevant to model the dynamics of $Y_t$.

We extracted unobserved factors from the "informational" time series included in $(N \times 1)$ vector of $X_t$ that consists of a balanced panel of 98 quarterly macroeconomic time-series. The number of informational time series is large, and must be greater than the number of factors and observed variables in the FAVAR system.

We estimate FAVAR here by implementing a two-step estimation (Bernanke et al. (2005)).

We assume that informational time series $X_t$ are related to the unobservable factors $F_t$ by the following observation equation.

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

where $F_t$ is a $k \times 1$ vector of common factors, $\Lambda^f$ is a $N \times k$ matrix of factor loadings, $\Lambda^y$ is $N \times M$, and $e_t$ are mean zero and normal, and assumed a small cross-correlation, which vanishes as $N$ goes to infinity.

In the first step, factors are obtained from the observation equation by imposing the orthogonality restriction $F'F/T = I$. This implies that $\hat{F}_t = \sqrt{T} \tilde{Z}$, where $\tilde{Z}$ is the eigenvectors corresponding to the $k$ largest eigenvalues of $XX'$, sorted in descending order. Stock and Watson (2002) showed that the factors can be consistently estimated by the first $r$ principles of components of $X$, even in the moderate changes in the loading matrix $\Lambda$. For this result to hold, it is important that the estimated number of factors, $k$, is larger than or equal to the true number, $r$. Because $N$ is sufficiently large, the factors are estimated precisely enough to be treated as data in subsequent regressions.

In the second step, we estimate the FAVAR equation, replacing $F_t$ by $\hat{F}_t$.

### 2.2 Data:

In our applications, $X_t$ consists of a balanced panel of 98 quarterly macroeconomic time series (Please see Appendix A for details). I took all data from INDB Finder PRO (IN information center). These series are transformed to eliminate unit roots and trends, and for that we took the logarithm and detrended with a one-sided Hodrick-Prescott (H-P) filter to most series. The data span the period from 1985Q3 to 2008Q1. Government investment ($\tilde{I}G$), government consumption ($\tilde{C}G$), inflation ($\tilde{\pi}$), private consumption ($\tilde{C}P$), labor hours ($\tilde{N}$), private investment ($\tilde{I}P$), output ($\tilde{Y}$), real wages ($\tilde{W}$), and nominal interest rate ($\tilde{i}$) are the observable factors $Y_t$ (See Appendix A for data on each).
2.3 Estimation:

We first estimated the following VAR model, and considered the extension of the baseline VAR model:

\[ Y_t = \sum_{i=1}^{2} A_i Y_{t-i} + u_t^Y \tag{3} \]

where \( Y_t = (\hat{Y}, \hat{CP}, \hat{IP}, \hat{CG}, \hat{IG}, \hat{W}, \hat{N}, \hat{\pi}, \hat{i}) \) to the following FAVAR framework.

\[
\begin{pmatrix}
Y_t \\
F_t
\end{pmatrix} = 
\begin{bmatrix}
\Phi_{11}(L) & \Phi_{12}(L) \\
\Phi_{21}(L) & \Phi_{22}(L)
\end{bmatrix} 
\begin{pmatrix}
Y_{t-1} \\
F_{t-1}
\end{pmatrix} + 
\begin{pmatrix}
u_t^Y \\
\nu_t^F
\end{pmatrix} \tag{4}
\]

\( Y_t = (\hat{Y}, \hat{CP}, \hat{IP}, \hat{CG}, \hat{IG}, \hat{W}, \hat{N}, \hat{\pi}, \hat{i}) \)

\( F_t = (F_{1t}, F_{2t}, F_{3t}) \)

where \( F_{1t}, F_{2t}, F_{3t} \) are the three factors we had from the following 1st step procedure.

In the first step, we make a regression \( X_t = \hat{\Lambda}^Y Y_t + \varepsilon_t \) firstly, and \( \hat{F}_t \) constructed from \( X_t - \hat{\Lambda}^Y Y_t \) by principal component method. This is because we employed recursive assumption (Choleski ordering) in the second step, and I put unobservable factor \( \hat{F}_t \) after the observable factors. This procedure prevent components of unobservable factors \( \hat{F}_t \) from responding contemporaneously to the observable factors \( Y_t \) and recursive assumption can avoid a criticism.

In the second step, we employed simple recursive frameworks. We ordered \( \hat{IG}, \hat{CG} \) first and treated its innovations as fiscal policy shocks in the standard way. This ordering imposes the identifying assumption that every other observable factor \( \hat{\pi}, \hat{CP}, \hat{N}, \hat{IP}, \hat{Y}, \hat{W}, \hat{i} \) respond to fiscal policy in the period. We estimate FAVAR by \( \hat{IG}, \hat{CG}, \hat{\pi}, \hat{CP}, \hat{N}, \hat{IP}, \hat{Y}, \hat{W}, \hat{i} \) ordering,\(^6\) and used three latent factors for FAVAR. We used two lags both in VAR and in FAVAR, and employing four lags led to similar impulse responses. Moreover, increasing the number of factors beyond this to five factors did not change the qualitative nature of our results.

2.4 Empirical Result:

Table 2.1 compares the VAR and FAVAR specifications for the vector \( Y_t = (\hat{IG}, \hat{CG}, \hat{\pi}, \hat{CP}, \hat{N}, \hat{IP}, \hat{Y}, \hat{W}, \hat{i}) \). We tested the restriction \( \Phi_{12}(L) = 0 \) by F-test. The results reported in Table 2.1 illustrate that factors are jointly significant (Granger cause) in most equations; the exceptions are in government
In the figure, IG: Government Investment, CG: Government Consumption, \( \pi \): Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate.

### Table 2.1. VAR and FAVAR specifications

<table>
<thead>
<tr>
<th>Equation</th>
<th>IG</th>
<th>CG</th>
<th>( \pi )</th>
<th>CP</th>
<th>N</th>
<th>IP</th>
<th>Y</th>
<th>W</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>0.53</td>
<td>0.44</td>
<td>0.16</td>
<td>0.46</td>
<td>0.68</td>
<td>0.85</td>
<td>0.75</td>
<td>0.54</td>
<td>0.95</td>
</tr>
<tr>
<td>S.E.</td>
<td>3.03</td>
<td>0.6</td>
<td>0.28</td>
<td>0.72</td>
<td>0.46</td>
<td>2.2</td>
<td>0.7</td>
<td>0.62</td>
<td>0.046</td>
</tr>
<tr>
<td>FAVAR</td>
<td>0.58</td>
<td>0.45</td>
<td>0.27</td>
<td>0.49</td>
<td>0.72</td>
<td>0.87</td>
<td>0.78</td>
<td>0.58</td>
<td>0.95</td>
</tr>
<tr>
<td>S.E.</td>
<td>2.85</td>
<td>0.59</td>
<td>0.26</td>
<td>0.7</td>
<td>0.43</td>
<td>2.03</td>
<td>0.66</td>
<td>0.59</td>
<td>0.045</td>
</tr>
<tr>
<td>( \chi^2(6) )</td>
<td>14.73</td>
<td>6.96</td>
<td>17.11</td>
<td>10.43</td>
<td>16.24</td>
<td>18.09</td>
<td>14.31</td>
<td>12.47</td>
<td>7.45</td>
</tr>
</tbody>
</table>

### Table 2.2. Residual Normality

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>5.99</td>
<td>14.82</td>
<td>13.72</td>
<td>0.28</td>
<td>0.39</td>
<td>0.41</td>
<td>0.65</td>
<td>6.67</td>
<td>13.77</td>
</tr>
<tr>
<td>FAVAR</td>
<td>0.97</td>
<td>24.78</td>
<td>4.01</td>
<td>1.49</td>
<td>0.05</td>
<td>0.05</td>
<td>0.01</td>
<td>0.52</td>
<td>13.23</td>
</tr>
<tr>
<td></td>
<td>0.62</td>
<td>0.000</td>
<td>0.13</td>
<td>0.47</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.77</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### Table 2.3. Serial correlation of residuals

<table>
<thead>
<tr>
<th>LM</th>
<th>LAG 1</th>
<th>LAG 2</th>
<th>LAG 3</th>
<th>LAG 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>( \chi^2 ) (81)</td>
<td>103.8</td>
<td>87.32</td>
<td>87.65</td>
</tr>
<tr>
<td>FAVAR</td>
<td>( \chi^2 ) (144)</td>
<td>152.02</td>
<td>163.12</td>
<td>148.17</td>
</tr>
</tbody>
</table>

In the figure, IG: Government Investment, CG: Government Consumption, \( \pi \): Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate.
consumption ($CG$), private consumption ($CP$), wages ($\bar{w}$), and interest rate ($\bar{i}$) equation.

Tables 2.2 and 2.3 report the evidence on the residual analysis from the VAR and FAVAR models. Table 2.2 contains the outcome of Jarque and Bera (1980) tests of null hypothesis of normality of residuals from each equation and for the joint 9 equation model. The null of normality is rejected for both VAR and FAVAR in the joint 9 equation model; the main cause of this rejection is the non-normality of residuals in $CG$ and $\bar{i}$ equations.

Table 2.3 reports the Breush-Godfrey Lagrange Multiplier tests of null hypothesis of no serial correlation of residuals at all lags from one to four. The result showed in the first lag of VAR specification points toward residual auto-correlations, while the null hypothesis of absence of residual correlation at any lags cannot be rejected in FAVAR specifications.

We then proceed to a further comparative analysis of the VAR and FAVAR models by considering impulse response function to fiscal policy shock. The results are shown in Figures 2.1 and 2.2. We plot responses for a horizon of 20 periods of quarterly $Y_t = (IG, CG, \bar{\bar{\pi}}, CP, N, IP, Y, W, \bar{i})$ series to a fiscal policy shock (government investment and government consumption) of both VAR and the FAVAR model.

Figure 1 shows the impulse response to one standard deviation government investment shock. The red line shows the response of the FAVAR model, and the blue line is that of the VAR model. The purple and green lines show 95% confidence intervals to the shocks of the VAR.

Figure 1 shows there are differences in impulse response between the VAR and FAVAR models. We can see that a government investment shock triggers an increase in private consumption (2nd row, 1st column) and output (3rd row, 1st column) by both the VAR and FAVAR models. The response of consumption and output are persistent, and reach their peak in the second period, and last for more than three years in both models. Wages (3rd row, 2nd column) also increase after a government investment shock for both models, while the increase is stronger in FAVAR specification. As to the response of private investment (2nd row, 3rd column), government investment crowds out private investment for all the time periods by VAR specification, while we can see strong crowds in for three years by the FAVAR model. We can also see nominal interest rate increase clearly from the FAVAR model (3nd row, 3rd column).

Figure 2 shows the impulse response to one standard deviation government consumption shock. Figure 2 also shows the differences in impulse response between the VAR and FAVAR models. We can see that a government consumption shock triggers an increase in private consumption (2nd row, 1st column) and output (3nd row, 1st column) both by models here as well. As for the response of private consumption, it very persistently increases and reaches its peak in the second period by the FAVAR model. VAR impulse has a larger response than the FAVAR impulse; it reaches its peak in fourth period. The response of

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6We followed Boaukez and Rebei (2007) for this VAR ordering.
Figure 1: Impulse response to IG impact with VAR-FAVAR comparison

Figure 1 shows impulse responses of 1std government investment shock. In the figure, IG: Government Investment, CG: Government Consumption, Pi: Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate. FAVAR (Red solid line): Impulse response of FAVAR, VAR (Blue solid line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.
Figure 2: Impulse response to CG impact with VAR-FAVAR comparison

Figure 2 shows impulse responses of 1std. government consumption shock. In the figure, IG: Government Investment, CG: Government Consumption, Pi: Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wages, i: Nominal Interest rate. FAVAR (Red solid line): Impulse response of FAVAR, VAR (Blue solid line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.
output has the same tendency. It reaches its peak in 3rd period and increase for 2 years by FAVAR model. The response of wages (3rd row, 2nd column) also seems to increase after a government consumption shock, but changes signs three times within the first four quarters for both models. As for the response of private investment (2nd row, 3rd column), a government consumption shock crowds out for the first 10 quarters by the FAVAR model, while we can see persistent crowds in movement from the VAR model.

3 The Model

The model we setup extends Bouakez and Rebei (2007) model in three dimension. First, we setup a New Keynesian model. Second, we include intertemporal investment adjustment cost and variable capital utilization as real rigidities. Third, we consider both government consumption and government investment and assumed government capital stocks enters the production function.

3.1 The Representative Household

The economy is populated by a single, infinitely lived representative agent that drives utility from effective consumption ($\tilde{C}_t$) and leisure ($1 - N_t$). Effective consumption is assumed to be the CES index of private consumption ($C^p$) and government consumption ($C^G$).

$$u(\tilde{C}_t, \tilde{C}_{t-1}N_t) = \frac{1}{1-\varepsilon} (\frac{\tilde{C}_t}{\tilde{C}_{t-1}^\gamma})^{1-\varepsilon} + \iota \ln(1 - N_t)$$

$$\tilde{C}_t = [\phi C^p_t \frac{1}{\varepsilon} + (1 - \phi) C^G_t \frac{1}{1-\gamma}]^{\frac{\varepsilon}{\varepsilon - 1}}$$

where $\varepsilon$ stands for the inverse of the long-run intertemporal elasticity of substitution, and $\varepsilon$ and $\iota$ are positive parameters. $\gamma \in (0, 1)$ measures the degree of habit formation. $v > 0$ is the elasticity of substitution between private consumption and government consumption.

The representative household supply labor and public capital to firms, and pays lump-sum tax to the government, $Z_t$.

$$C^p_t + I^p_t + \psi(U_t)K^p_t + \frac{B_t}{P_t} + Z_t \leq w_t N_t + r_t U_t K^p_t + \frac{B_{t-1}R_{t-1}}{P_t} + \Xi_t$$

where $B_t$ is government debt issued at $t$, which pays $B_t R_t$ units of goods at $t+1$, $\Xi_t$ is dividends from owning imperfect competitive intermediate goods firm, $U_t$.

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7Earlier studies that assume consumer preferences to be CES index of private consumption and government spending include Amano and Wirjanto (1997), Okubo (2003), Linnenmann and Schabert (2003), Boualez and Rebei (2007), Marrattin and Marzo (2010), Mazraani (2010).

8We assumed habit formation for effective consumption following Boualez and Rebei (2007).
is the capital utilization rate, and $r_tU_tK_t^p$ represents household earnings from supplying capital services. The steady state capital utilization rate is assumed to be $U = 1$. Increases from steady state utilization incurs a cost, $\psi(U_t)K_t^p$, and we assume functional form for the adjustment cost of capital utilization is given by

$$\psi(U_t) = \frac{U_t^{1+\psi^{-1}} - 1}{1 + \psi^{-1}}$$

where $\psi$ is the inverse of elasticity of the capital utilization with respect to the rental rate of one effective unit of private capital and $\psi > 0$. Note that $\psi(U_t)$ is an increasing, convex function ($\psi'(U_t) > 0$, $\psi''(U_t) > 0$), and in steady states, cost is zero ($\psi(1) = 0$). $U_tK_t^p$ represents the effective units of private capital. The rental rate of one effective unit of private capital is $r_t$ and wage rate is $w_t$. Private capital evolves according to

$$K_{t+1}^p = (1 - \delta^p)K_t^p + \{1 - S(\frac{I_t^p}{P_t})\}I_t^p$$  \hspace{1cm} (8)

where $\delta^p$ is the private capital depreciation rate, and $S(\bullet)$ stands for adjustment cost for capital defined as a quadratic function as follows.

$$S(\frac{I_t^p}{P_{t-1}}) = \frac{1}{2\xi} (\frac{I_t^p}{P_{t-1}} - 1)^2$$  \hspace{1cm} (9)

where $\xi$ implies inverse of the elasticity of investment on the price of capital. As can be seen from the above specification, the bigger the deviation of the current physical investment from the previous period, the higher the adjustment cost. Therefore, it will be in the interest of the household to install the capital as smoothly as possible to minimize the leakage. Also, it should be noted that adjustment cost is zero in steady states, i.e. $S(1) = S'(1) = 0$.

Given the budget constraint (7) and capital accumulation equation (8), the dynamic optimizing problem of the household is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, \tilde{C}_{t-1}N_t)$$

$$-\lambda_t\{C_t^p + I_t^p + \psi(U_t)K_t^p + \frac{B_t}{P_t} + Z_t - w_tN_t - r_tU_tK_t^p - \frac{B_{t-1}R_{t-1}}{P_t} - \frac{P_t}{P_t}\}$$

$$-\mu_t\{K_{t+1}^p - (1 - \delta^p)K_t^p - (1 - \frac{1}{2\xi}(\frac{I_t^p}{P_{t-1}} - 1)^2)I_t^p\}]$$

where $\lambda_t$ stands for the Lagrange multiplier attached to the budget constraint.

---

We introduce variable capital utilization as in Christiano, Eichenbaum, and Evans (2005), in which households are assumed to act as investor as well and make capital accumulation and utilization decisions. This assumption is made for convenience; at the cost of more complicated notation, they could work with an alternative decentralization scheme in which firms make these decisions.
at point $t$ and $\mu_t$ stands for the Lagrange multiplier attached to the capital accumulation equation.

The first order conditions associated with each control variables, $B_t, I_t^p, N_t, C_t^p, K_{t+1}^p, U_t$ are

bond holdings:

$$\lambda_t = \beta E_t[\lambda_{t+1} R_t \frac{P_t}{P_{t+1}}]$$  \hspace{1cm} (11)

physical investment:

$$\lambda_t = \mu_t [1 - \frac{1}{2\xi} - \frac{3}{2\xi} \left( \frac{I_t^p}{P_{t-1}} \right)^2 + \frac{2}{\xi} \left( \frac{I_t^p}{P_{t-1}} \right) - \beta E_t \mu_{t+1} \left( \frac{I_{t+1}^p}{P_{t+1}} \right)^2 (1 - \frac{P_{t+1}}{P_t})]$$  \hspace{1cm} (12)

labor supply:

$$\lambda_t = \frac{\ell}{w_t(1 - N_t)}$$  \hspace{1cm} (13)

consumption:

$$\lambda_t = \phi \left( \frac{\bar{C}_t}{C_t^p} \right)^{\frac{\gamma}{\gamma - 1}} \left\{ \left( \frac{1}{C_{t-1}^\gamma} \right) \left( \frac{\bar{C}_t}{C_{t-1}^\gamma} \right)^{-\gamma} - \beta \gamma E_t \left[ \frac{\bar{C}_{t+1}}{C_{t+1}^\gamma} \left( \frac{\bar{C}_{t+1}}{C_t^\gamma} \right)^{-\gamma} \right] \right\}$$  \hspace{1cm} (14)

capital holdings:

$$\mu_t = \beta E_t [\lambda_{t+1} \{ r_{t+1} U_{t+1} - \psi(U_{t+1}) \} + \mu_{t+1} (1 - \delta^p)]$$  \hspace{1cm} (15)

capital utilization:

$$r_t = \psi'(U_t)$$  \hspace{1cm} (16)

As to the first order condition with the physical investment (12), the LHS can be interpreted as the marginal cost of investment. By investing in one additional consumption good, the household forgo the same amount of consumption goods from its budget. RHS of the equation (12) represents the marginal benefit of investment. By investing in one additional unit, the household can increase the amount of private capital stock to some extent, but the magnitude of increase in capital stock is reduced to the leakage in capital installment. In the first bracket in (12), this leakage of capital installment on the margin is represented. In addition, we set the adjustment cost function as (9), a marginal change of the current investment will also affect the next period’s adjustment cost, and this effect is represented in the second bracket. Both these effects multiplying with
the shadow price of capital will constitute the marginal benefit from additional investment.

Moreover, as to the first order condition with private consumption (14), the LHS is marginal cost of private consumption. The RHS of the equation (14) represents the marginal utility of private consumption. \( \phi(\frac{C_t}{C_{t-1}})^{\gamma} \) represents additional effective consumption by one unit of private consumption, and the second bracket represents marginal utility from one unit of effective consumption. Notice that the second bracket reduces to \( C_t^{-\gamma} \) only if current effective consumption matters (\( \gamma = 0 \)).

Next, equation (15) represents the first order condition associated with the capital holdings. LHS \( \mu_t \) is the shadow price of capital and the marginal cost of adding one unit of capital at time \( t \). The RHS represents a marginal benefit of adding one unit of capital. Households can expect to increase \((1 - \beta^p)\) unit of capital at time \( t + 1 \), thus \( \beta \mu_{t+1}(1 - \beta^p) \) represents a present value marginal benefit of adding one unit of capital, and \( r_{t+1}U_{t+1} - \psi(U_{t+1}) \) represents household’s additional income via capital holding minus additional capital utilization cost coming with the additional capital lending.

Finally, as can be seen from (16), the optimality condition regarding capital utilization requires households to equalize the marginal cost of capital utilization to the rental rate. By increasing the capital utilization level marginally, the household can increase its income by \( r_t K^p \), which can be considered a marginal benefit to the household. However, an increase in capital utilization delivers a cost. By increasing the capital utilization, the household need to give up \( \psi'(U_t) \) amount of consumption goods on its margin.

3.2 Firms

3.2.1 The final goods firms

We assume that there is a continuum of intermediate firms of unit mass indexed by \( i \in [0, 1] \) and each firm produces an intermediate good that is different from that of other firms. The continuum of intermediate goods in period \( t \), \( Y_{i,t} \) gets bundled by final goods firms into final goods \( Y_t \). The final goods production technology is

\[
Y_t = \left[ \int_0^1 Y_{i,t}^{\eta - 1} di \right]^{\frac{1}{\eta - 1}}
\]

(17)

where \( \eta \) is the elasticity of substitution in production and governing the firm’s markup over the marginal cost.

A profit-maximizing final goods firm chooses the amount of intermediate goods to maximize profit given aggregate price \( P_t \) and intermediate goods price \( P_{i,t} \). We could have a demand function of goods \( i \) as,

\[
Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t
\]

(18)
Putting this demand for sector i’s output (3.14) into the bundler function (3.13), we can have a final goods pricing rule of,
\[ P_t = \left( \int_0^1 P_{i,t}^{1-\eta} d\eta \right)^{\frac{1}{1-\eta}} \]  
(19)

3.2.2 The intermediate goods firms

The production function for intermediate goods firm i is
\[ Y_{i,t} = A_t K_i^{P_i} N_{i,t} \left( 1 - \alpha \right) (K_t^G)^{\mu} \]  
(20)
where \( K_i^{P_i} = U_{i,t} K_i^{P_i} \) and all firms are subject to the same technology shock, \( A_t \).

\( K_t^G \) is the government capital stock. Notice that \( K_t^G \) does not have subscript \( i \), thus we assume \( K_t^G \) as well has common productivity to each firm. This production function is increasing return to scale as a whole but private sector resource is a constant return to scale\(^\dagger\).

Under a Calvo (1983)-type sticky price setting, for any given period \( t \), each firm has a \( \rho \) probability that it will keep the price of the previous period, and \( 1 - \rho \) probability that it will be able to choose its price optimally. An intermediate goods firm, \( i \), which can choose price in period \( t \), choose the price, \( P_{i,t}^* \), to maximize profit:
\[ \max_{P_{i,t}^*} E_t \sum_{j=0}^{\infty} \beta^j \rho^j \left[ P_{i,t+j}^* Y_{i,t+j} \left( \frac{P_{t+j} + r_{t+j}}{P_{t+j}^{P_i}} \right)^{\eta} - P_{t+j} r_{t+j} K_i^{P_i} N_{i,t+j} \right] \]  
(21)
\[ s.t. Y_{i,t+j} \left( \frac{P_{t+j} + r_{t+j}}{P_{t+j}^{P_i}} \right)^{\eta} = A_t K_i^{P_i} N_{i,t} \left( 1 - \alpha \right) (K_t^G)^{\mu} \]  
(22)

A firm that is maximizing profit is simultaneously minimizing total cost. The cost minimization problem for the firm \( i \) can be expressed as follows.
\[ \text{cost function: } \min_{K_i^{P_i}, N_{i,t}} r_t K_i^{P_i} + w_t N_{i,t} + mc_{i,t}(Y_{i,t} - A_t K_i^{P_i} N_{i,t} \left( 1 - \alpha \right) (K_t^G)^{\mu}) \]  
(23)
where the Lagrange multiplier, \( mc_{i,t} \), is the marginal cost of producing \( Y_{i,t} \).

Solving the cost minimization, the first order condition is
\[ \frac{w_t}{r_t} = \frac{(1 - \alpha) K_i^{P_i}}{\alpha N_{i,t}} \]  
(24)
\(^\dagger\)As for production function, Mitsui and Inoue (1995) showed that, in Japan, assuming a constant return to scale for the private sector is better than assuming a constant return to scale as a whole (including productive public capital) from a production function estimation using prefectural data.
solve (24) for $\bar{K}_{i,t}^p$ and $N_{i,t}$, and substituting these factor demands into the cost equations,

$$TC_{i,t} = r_t K_{i,t}^p + w_t N_{i,t} = \frac{Y_{i,t}}{A_t} \left(1 + \alpha \right) \frac{1}{\alpha^\alpha \frac{w_t^{1-\alpha}}{r_t^{\alpha}} \bar{K}_{i,t}^p}$$

so, the firm $i$’s marginal cost is

$$mc_{i,t} = \frac{TC_{i,t}}{Y_{i,t}} = \frac{1}{A_t} \left(1 + \alpha \right) \frac{1}{\alpha^\alpha \frac{w_t^{1-\alpha}}{r_t^{\alpha}} \bar{K}_{i,t}^p}$$

Notice that the specification of marginal cost (26) does not depend on subscript $i$, and this implies the marginal cost is symmetric across firms. Since the marginal cost is symmetric across firms, we simply suppress subscript $i$ on marginal cost.

Substituting total costs (25) to profit maximization problems (21) yields,

$$\max_{P_{i,t}} \sum_{j=0}^{\infty} \beta^j \rho^j [P_{i,t} Y_{i,t+j} (\frac{P_{i,t}^*}{P_{i,t}})^\eta - P_{t+j} r_t + \bar{K}_{i,t+j}^p - P_{t+j} w_{t+j} N_{i,t+j}]$$

$$= E_t \sum_{j=0}^{\infty} (\beta \rho)^j [P_{i,t}^* - P_{i+j} mc_{i+j}] Y_{i,t+j}$$

The first order condition for the profit maximization problem yields

$$E_t \sum_{j=0}^{\infty} (\beta \rho)^j Y_{i,t+j} \left[1 - \eta + \eta \frac{P_{i+j}^*}{P_{i,t}^*} mc_{i+j} \right] = 0$$

Rearranging further yields the following optimal pricing rule for firm $i$.

$$P_{i,t}^* = \frac{\eta}{\eta - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta \rho)^j Y_{i,t+j} mc_{i+j}}{E_t \sum_{j=0}^{\infty} (\beta \rho)^j Y_{i,t+j}}$$

Notice that $\eta > 1$, and the expression $\frac{\eta}{\eta - 1} > 1$ is the gross markup of the intermediate goods firm $i$’s price over the ratio of the discounted stream of nominal total costs divided by the discounted stream of real output.

Notice that all intermediate goods firms that can fix their prices set the same markup over the same marginal cost, so in every period $t$, $P_{i,t}^*$ is the same for all $1 - \rho$ firms that adjust their prices, and all non-adjusting firms keep their price as it was in the previous period. Combining with the final goods pricing rule (19), we have the aggregate goods pricing rule,

$$P_{t}^{1-\eta} = \rho P_{t-1}^{1-\eta} + (1 - \rho) P_{t}^{1-\eta}$$
3.3 The Policy Side

3.3.1 Fiscal Authority

The fiscal authority purchases final goods $C^G_t$, $I^G_t$, issues bonds $B_t$, and levies lump-sum tax $Z_t$. The flow budget constraint for fiscal authority is,

$$C^G_t + I^G_t + \frac{R_{t-1}B_{t-1}}{P_t} = Z_t + \frac{B_t}{P_t}$$ (30)

Government spending $C^G_t$, $I^G_t$ evolve according to an AR(1) stochastic process.

$$\hat{C}^G_t = \rho_{CG}\hat{C}^G_{t-1} + \epsilon^G_t$$ (31)
$$\hat{I}^G_t = \rho_{IG}\hat{I}^G_{t-1} + \epsilon^G_t$$ (32)

government capital stock evolve according to

$$K^G_{t+1} = I^G_t + (1 - \delta^G)K^G_t$$ (33)

where $\delta^G$ is the government capital depreciation rate.

3.3.2 Monetary Policy

The central bank sets the nominal interest rate according to a simple feedback rule of the following rule11.

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1 - \rho_r)\phi_{\pi\pi}\pi_{t-1} + (1 - \rho_r)\phi_{\pi Y}\hat{Y}_t$$ (34)

where $\pi_{t-1} \equiv \log(P_{t-1}/P_{t-2})$ denote inflation rate.

3.4 Market Clearing Condition

We impose the market-clearing condition for the final goods market. We require the supply of final goods to be equal to the demand of final goods for private consumption, private investment, capital utilization, government consumption, and government investment12.

$$Y_t = C^P_t + I^P_t + \psi(U_t)K^P_t + C^G_t + I^G_t$$ (35)

11This type of Taylor rule with interest-rate inertia can be found in Rotemberg and Woodford (1999), Clarida, Gali, and Gertler (1999), and Christiano, Eichenbaum, and Evans (2005).

12A Log-linearized version of the model could be found in Appendix B.
4 Model Estimation

4.1 Estimation Methodology

The purpose of this section is to use Japanese data to obtain values for the model parameters. In particular, we are interested in measuring the extent of complementarity between government consumption and private consumption and public capital share in the production function. We describe below our estimation methodology briefly following Boaukez and Rebei (2007) and Iiboshi, Nishiyama and Watanabe (2008).

The model's solution can be written in the following recursive equilibrium low of motion:

\[ s_t = G(\theta) s_{t-1} + H(\theta) \varepsilon_t \] (36)

where \( s_t \) is a 23 \times 1 vector of endogenous variables: \( s_t = [\widehat{Y}_t, \widehat{\pi}_t, \widehat{C}_t, \widehat{C}_G^t, \widehat{I}_t, \widehat{\mu}_t, \widehat{\pi}_t, \widehat{m}_t, \widehat{K}_t, \widehat{K}_G^{t+1}, \widehat{R}_t, \widehat{\alpha}_t, \widehat{I}_G^t, \widehat{C}_G^t, \widehat{E}_t \pi_{t+1}, \widehat{E}_t C_{t+1}, E_{t+1} \lambda_{t+1}, E_{t+1} \mu_{t+1}, E_{t+1} I_{t+1}, E_{t+1} I_{G+1}, \varepsilon_t] \), and \( \varepsilon_t \) is a vector of endogenous shocks: \( \varepsilon_t = [\varepsilon_t^A, \varepsilon_t^I, \varepsilon_t^{CG}] \), and \( \theta \) is the vector of deep parameters to be estimated. From equation (4.1), we set a state space model which consists of a transition equation and measurement equation as follows;

\[ s_t = G(\theta) s_{t-1} + H(\theta) \varepsilon_t \] (37)

\[ y_t = J s_t \] (38)

where \( y_t \) is the 9 \times 1 vector of observable variables at time \( t \) and \( J \) is a 9 \times 23 matrix that links the observed \( y_t \) vector and the unobserved \( s_t \). For this state space model with Gaussian error terms, unobservable variables \( s_t \) and the likelihood of the model are obtained using a Kalman filter. The Kalman filter is the algorithm that provides the mean and the covariance matrix of the state vector \( s_t \) (\( t = 1, ..., T \)) conditional on the observations up to \( t \), i.e., \( (Y_1, ..., Y_T) \) in a linear Gaussian state space model.

A crucial requirement of the Kalman filter is that the number of observable variables used in the estimation does not exceed the number of shocks in the model; otherwise, the variance-covariance matrix of the residuals becomes singular. In our case, we have only three structural shocks, so we can use as little as three series. To circumvent this problem, we followed Boaukez and Rebei (2007) and add measurement errors to the variables in the measurement equation\(^{13}\)

\[ s_t = G(\theta) s_{t-1} + H(\theta) \varepsilon_t \] (39)

\[ y_t = J s_t + \zeta_t \] (40)

\(^{13}\)The addition of measurement errors to get around the singularity problem has been done by McGrattan, Rogerson and Wright (1997), Ireland (2004), and Boaukez and Rebei (2007), Nishiyama et al (2011).
Table 4.1 Value of Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meanings</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>Weight of Private spending in effective consumption</td>
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<tr>
<td>( \alpha )</td>
<td>Elasticity of output with respect to private capital</td>
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<td>( \beta )</td>
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<td>( \delta_G )</td>
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<td>( \eta )</td>
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<tr>
<td>Steady-State Value</td>
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<tr>
<td>( N )</td>
<td>Fraction of Time worked</td>
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<tr>
<td>( CG/Y )</td>
<td>Government consumption-to-output ratio</td>
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</tr>
<tr>
<td>( IG/Y )</td>
<td>Government investment-to-output ratio</td>
<td>0.05</td>
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</table>

where \( \varepsilon_t \sim i.i.d. N(0, Q(\theta)) \), \( \zeta_t \) is the vector of measurement errors, and \( \zeta_t \sim i.i.d. N(0, R) \), \( E(\varepsilon_t, \zeta_t) = 0 \).

For estimation, we used DYNARE, which is a convenient tool for conducting Bayesian estimations. The DYNARE toolbox derives reduced-form representation of the model and automatically provides stability and eigenvalue analysis. In addition, it enables us to conduct Bayesian estimations\(^{14}\). The series used in the estimation are output, private consumption, private investment, government consumption, government investment, real wages, labor hours, nominal interest rate, and inflation rate. We used the common data with Section 2, but we limited the data period from 1985Q3 to 1998Q4 because of the zero-interest-rate policy started in February 1999 in Japan\(^{15}\).

4.2 Priors and Calibrated Value

Table 4.1 presents the values assigned to the calibrated parameters. We fix most parameters following Boaukez and Rebei (2007); \( \alpha = 0.36 \), which implies a labor share of 64%, \( \beta = 0.99 \), which implies an annual steady state interest rate of 4%, \( \delta_P = 0.025 \), which puts the annual steady state depreciation of private capital at 10%, \( \delta_G = 0.01 \), which means the annual steady state depreciation of public capital is 4%. The price elasticity of demand \( \eta \) is set equal to 6 to be consistent with a steady state net markup of 20 percent. A preliminary attempt to estimate \( \phi \) turned out to be poorly identifiable, so we set the same calibrated

\(^{14}\)DYNARE firstly evaluate likelihood using Kalman filter by maximum likelihood estimates. Then DYNARE finds the mode of posterior distribution and estimate posterior distribution of parameters of our interest. The posterior distribution is given by a nonlinear and complicated function of deep parameters, so DYNARE resorts to sampling-like methods, a Metropolis-Hastings algorithm.

\(^{15}\)Zero- Lower band of nominal interest rate brings model nonlinearity, which is a difficult task. We skip here this problem by limiting our data period. Doi et al. (2011) showed monetary policy response in Japan (Taylor rule coefficient on inflation or output gap) have been changed with high probability in the mid 1990s by estimation of monetary policy function with Markov switching.
value of 0.8 as Bouakez and Rebei (2007) did, but we could estimate parameter \( \varepsilon \) because we included nominal interest rate and inflation rate as observed data while Bouakez and Rebei (2007) did not. We also set steady state value at \( N = 0.31, \frac{CG}{Y} = 0.15, \frac{IG}{Y} = 0.05 \) following Bouakez and Rebei (2007).

Table 4.2 reports the prior distribution of the other estimated parameters. For the choice of prior distribution, the means were set at values that correspond with other studies, and standard errors were set so that the domain covers a reasonable range of parameter values, including values estimated by previous studies. We mainly made reference to Bouakez and Rebei (2007), but we also made reference to Iiboshi, Nishiyama, and Watanabe (2008), Iwata (2010) and Sugo and Ueda (2008), all of which estimated a DSGE model for the Japanese data.

We set mean of \( \varepsilon = 2 \), which is the value given in Bouakez and Rebei (2007),

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<th>meanings</th>
<th>type</th>
<th>mean</th>
<th>s.e.</th>
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</tr>
<tr>
<td>Standard Errors for Measurement Errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta_{\gamma} )</td>
<td>SE of measurement err. for output gap</td>
<td>inv. Gamma</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\omega} )</td>
<td>SE of measurement err. for private consumption</td>
<td>inv. Gamma</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\mu} )</td>
<td>SE of measurement err. for private investment</td>
<td>inv. Gamma</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \eta_{\gamma} )</td>
<td>SE of measurement err. for government consumption</td>
<td>inv. Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\alpha} )</td>
<td>SE of measurement err. for government investment</td>
<td>inv. Gamma</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>( \eta_{u} )</td>
<td>SE of measurement err. for real wage</td>
<td>inv. Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\pi} )</td>
<td>SE of measurement err. for real wage</td>
<td>inv. Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\pi} )</td>
<td>SE of measurement err. for inflation</td>
<td>inv. Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_{\pi} )</td>
<td>SE of measurement err. for nominal interest rate</td>
<td>inv. Gamma</td>
<td>0.05</td>
<td>1</td>
</tr>
</tbody>
</table>
and we took a wide standard error 2 for the parameter. We set the mean at \( \nu = 0.8 \), which implies private and government consumption are substitutes \((\frac{1}{\nu} - \varepsilon < 0)\) in the prior mean, but we took a standard error of \( \nu = 0.5 \), which covers the range of estimated value of \( \nu \) in Bouakez and Rebei (2007) 0.3. \( \gamma, \xi, \rho \) could be seen in ordinary range and similar to previous studies. As to \( \Psi \), there seems to be wide difference in prior setting in previous studies. Smets and Wouters (2003), which studies the euro zone, set its mean at 0.2 and its standard error at 0.075, while Onatski and Williams (2005), which studies U.S. data, set uniform distribution between 2.8 – 10. Here, we set prior mean and standard error of \( \Psi \) to 1. As to the parameter of productivity of public capital, \( \mu \), we made reference to Aschauer (1989) 0.36, 0.25 by Mitsui and Inoue (1995), or 0.20 by Kawaguchi et al. (2005). We made reference to Iwata (2011) for prior of policy parameters and standard errors of shocks and to Sugo and Ueda (2008) for prior of shock persistence.

Given the prior distributions, DYNARE calculates the posterior distributions using a Metropolis-Hastings Markov chain Monte Carlo (MCMC) algorithm. We sample 1,000,000 periods each, discarding the first 500,000 periods.

4.3 Results: Estimated Parameters

Table 4.3 reports the posterior means and 90% Confidence Intervals (CI) for the parameters for 2 models estimated; "baseline model" means a model without public capital (the public capital productivity parameter \( \mu \) is set to 0), and "public capital in PF" means public capital is in production function (\( \mu \) is estimated). Several important parameters are worth commenting comparing with Bouakez and Rebei (2007). (We mostly comment on Baseline model below.)

The parameters for elasticity of substitution \( \nu \) is 0.56 and the parameter of inverse of intertemporal substitution of consumption \( \varepsilon \) is 0.95, so \( \frac{1}{\nu} - \varepsilon > 0 \). This means the necessary condition for government spending to increase the marginal utility of consumption is satisfied, and private and government consumption are Edgeworth complements. This result is the same with Bouakez and Rebei (2007) for U.S. data; they had \( \nu = 0.33 \) (and \( \varepsilon \) is calibrated value 2). An earlier study by Okubo (2003), which analyzed data in Japan and used a partial equilibrium approach based on Euler equation, estimated \( \nu = 1.39 \) and \( \varepsilon = 0.19 \cdot 0.61 \) (so \( \frac{1}{\nu} - \varepsilon = 0.12 \cdot 0.53 > 0 \)) and concluded that private consumption and government consumption are complements or unrelated, so this is also in the same line.

---

16This is the same prior mean and standard error of \( \Psi \) used by Sugo and Ueda(2008) although the specification of capital utilization is different.
17Mitsui and Inoue (1995) estimated production function using macroeconomic time series and found the marginal product of public capital to be around 0.25. Kawaguchi et al. (2005) estimated marginal product of public capital using prefecture-level data and exogenous variation in number of seats in the Diet by electoral reform in 1994 as an instrument variable. They found the elasticity to be around 0.2, although the effect is not precisely estimated.
18We show model implications using calibrations in Appendix C.
19Iwata (2012) concluded the relationship between private and public consumption may be complements using Japanese data, although the definition of effective consumption is different.
Table 4.3. Posterior distributions for the models estimated

<table>
<thead>
<tr>
<th>parameters</th>
<th>Baseline Model (mean)</th>
<th>Baseline Model (CI(Low))</th>
<th>Baseline Model (CI(High))</th>
<th>Public Capital in PF (mean)</th>
<th>Public Capital in PF (CI(Low))</th>
<th>Public Capital in PF (CI(High))</th>
<th>Bouakez and Rebei (2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>structual parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>υ</td>
<td>0.557</td>
<td>0.389</td>
<td>0.714</td>
<td>0.589</td>
<td>0.372</td>
<td>0.782</td>
<td>0.332</td>
</tr>
<tr>
<td>ε</td>
<td>0.950</td>
<td>0.510</td>
<td>1.353</td>
<td>0.982</td>
<td>0.562</td>
<td>1.361</td>
<td>2(calibrated value)</td>
</tr>
<tr>
<td>γ</td>
<td>0.463</td>
<td>0.170</td>
<td>0.796</td>
<td>0.497</td>
<td>0.188</td>
<td>0.810</td>
<td>0.259</td>
</tr>
<tr>
<td>ξ</td>
<td>1.684</td>
<td>1.148</td>
<td>2.210</td>
<td>1.656</td>
<td>1.126</td>
<td>2.180</td>
<td>1.052</td>
</tr>
<tr>
<td>Ψ</td>
<td>2.109</td>
<td>0.867</td>
<td>3.328</td>
<td>2.062</td>
<td>0.827</td>
<td>3.291</td>
<td>N.A</td>
</tr>
<tr>
<td>ρ</td>
<td>0.605</td>
<td>0.441</td>
<td>0.755</td>
<td>0.601</td>
<td>0.440</td>
<td>0.765</td>
<td>N.A</td>
</tr>
<tr>
<td>μ</td>
<td>N.A</td>
<td>N.A</td>
<td>N.A</td>
<td>0.295</td>
<td>0.108</td>
<td>0.484</td>
<td>N.A</td>
</tr>
<tr>
<td>Policy parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_r</td>
<td>0.772</td>
<td>0.625</td>
<td>0.923</td>
<td>0.772</td>
<td>0.624</td>
<td>0.924</td>
<td>N.A</td>
</tr>
<tr>
<td>ρ_π</td>
<td>3.015</td>
<td>1.880</td>
<td>4.083</td>
<td>3.010</td>
<td>1.928</td>
<td>4.111</td>
<td>N.A</td>
</tr>
<tr>
<td>ρ_y</td>
<td>0.053</td>
<td>0.004</td>
<td>0.101</td>
<td>0.055</td>
<td>0.005</td>
<td>0.104</td>
<td>N.A</td>
</tr>
<tr>
<td>Shock persistence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_a</td>
<td>0.820</td>
<td>0.743</td>
<td>0.899</td>
<td>0.820</td>
<td>0.740</td>
<td>0.900</td>
<td>0.873</td>
</tr>
<tr>
<td>ρ_cg</td>
<td>0.727</td>
<td>0.565</td>
<td>0.888</td>
<td>0.727</td>
<td>0.559</td>
<td>0.896</td>
<td>0.912</td>
</tr>
<tr>
<td>ρ_IG</td>
<td>0.891</td>
<td>0.846</td>
<td>0.937</td>
<td>0.903</td>
<td>0.855</td>
<td>0.951</td>
<td>N.A</td>
</tr>
<tr>
<td>Standard Errors for Structural Shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>η_a</td>
<td>0.377</td>
<td>0.094</td>
<td>0.699</td>
<td>0.356</td>
<td>0.095</td>
<td>0.674</td>
<td>0.005</td>
</tr>
<tr>
<td>η_cg</td>
<td>0.181</td>
<td>0.047</td>
<td>0.334</td>
<td>0.220</td>
<td>0.045</td>
<td>0.372</td>
<td>0.013</td>
</tr>
<tr>
<td>η_IG</td>
<td>0.282</td>
<td>0.068</td>
<td>0.537</td>
<td>0.250</td>
<td>0.070</td>
<td>0.466</td>
<td>N.A</td>
</tr>
<tr>
<td>Standard Errors for Measurement Errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>η_r</td>
<td>0.327</td>
<td>0.042</td>
<td>0.509</td>
<td>0.188</td>
<td>0.045</td>
<td>0.360</td>
<td>-</td>
</tr>
<tr>
<td>η_π</td>
<td>0.178</td>
<td>0.046</td>
<td>0.323</td>
<td>0.213</td>
<td>0.046</td>
<td>0.360</td>
<td>-</td>
</tr>
<tr>
<td>η_p</td>
<td>0.875</td>
<td>0.242</td>
<td>1.604</td>
<td>0.915</td>
<td>0.234</td>
<td>1.712</td>
<td>-</td>
</tr>
<tr>
<td>η_a</td>
<td>0.095</td>
<td>0.022</td>
<td>0.172</td>
<td>0.106</td>
<td>0.022</td>
<td>0.188</td>
<td>-</td>
</tr>
<tr>
<td>η_IG</td>
<td>0.484</td>
<td>0.111</td>
<td>0.920</td>
<td>0.503</td>
<td>0.111</td>
<td>0.935</td>
<td>-</td>
</tr>
<tr>
<td>η_cg</td>
<td>0.088</td>
<td>0.023</td>
<td>0.163</td>
<td>0.090</td>
<td>0.023</td>
<td>0.169</td>
<td>-</td>
</tr>
<tr>
<td>η_w</td>
<td>0.086</td>
<td>0.023</td>
<td>0.158</td>
<td>0.091</td>
<td>0.024</td>
<td>0.164</td>
<td>-</td>
</tr>
<tr>
<td>η_i</td>
<td>0.098</td>
<td>0.022</td>
<td>0.179</td>
<td>0.094</td>
<td>0.023</td>
<td>0.172</td>
<td>-</td>
</tr>
<tr>
<td>η</td>
<td>0.051</td>
<td>0.011</td>
<td>0.097</td>
<td>0.056</td>
<td>0.011</td>
<td>0.103</td>
<td>-</td>
</tr>
</tbody>
</table>
The parameter for consumption habit formation $\gamma$ is about 0.48, which is larger than Bouakez and Rebei (2007) 0.25. The parameter for capital utilization cost, $\Psi$, is 2.1 and this is similar to Ouaknine and Williams (2004) 2.8. The parameter of inverse of investment adjustment costs, $\xi$ is 1.7. This implies that investment increases 1.7 percent in the long run following a 1 percent increase in Tobin’s $q$. This estimated parameter is larger than that found in other studies, i.e. Levin et al. (2005) 0.55 or Smets and Wouters (2007) 0.14.20

Regarding inflation dynamics, the Calvo price-setting parameter $\rho$ is 0.61, so the probability that a given price can be optimized in a quarterly period $(1 - \rho)$ is 0.39. This implies an average contract duration of price setting is estimated to be about 2.5 quarters.

Regarding monetary policy parameters, the coefficient on lagged interest rates in the monetary policy rule $\rho_r$ is 0.82. This implies that monetary policy has high inertia. The response of interest rate to inflation $\rho_{\pi}$ is 3, which is much greater than one, indicates that the monetary authority in Japan reacts very actively to inflation. On the contrary, the response to output, $\rho_Y$ is small, at 0.05.

As for the results of "public capital in production function," our interest is in the parameter of productivity of public capital in production function, and it is 0.30. This is smaller than Aschauer’s (1989) 0.36 but larger than the estimate of 0.25 by Mitsui and Inoue (1995), 0.2 by Kawaguchi et al. (2005), or 0.046 by Iwata (2012). Other parameters estimated are similar to the baseline case, so we omit an explanation of these results here.

4.4 Impulse Response

In this subsection, we would like to see the impulse responses of endogenous variables to government investment and government consumption shocks for five years following each of the shocks. The horizontal axis represents time on a quarterly scale, and the vertical axis represents percentage deviation from equilibrium.

Figure 3 illustrates impulse responses to a government consumption shock. This captures important channels that determine impact on a government consumption shock: crowding-out effects, wealth effects, and the effects from private and government consumption Edgeworth complementarity.

First, higher government consumption absorbs existing resources so that there are fewer goods available for the private sector to save (invest) or consume. As goods today become more valuable and not all intermediate goods firm can adjust its price, the real marginal cost (rental cost of private capital and real wages) increases, but the increase of capital utilization alleviate the surge of the rental cost of private capital. Second, higher government consumption financed by lump-sum tax generates a negative wealth effect, encouraging agents from us here (Iwata (2012) used a linear form, $C_t = C_t^P + \varepsilon C_t^G$).

20. Although Bouakez and Rebei (2007) estimated under capital adjustment cost setting, the parameter for adjustment cost is 1.05, and is smaller than ours.
Figure 3: Impulse responses of estimated DSGE model (Baseline Model) (government consumption shock)


Blue Solid line: Impulse responses, Green and Red dotted Lines: Upper and Lower bounds of 90% HPD interval.
to work harder, and this will increase output but reduce marginal productivity of labor on the other hand. Third, when private and government consumption are Edgeworth complements, government consumption increases marginal utility of consumption, so people consume more in the current period. On the other hand this leads to increase labor supply and real wages suffer decreasing effects.

To see the impulse responses, as to private consumption, the effect of Edgeworth complementarity dominates crowding-out effects from our parameters estimated. As to real wages, the increase of real marginal cost dominate for short and it increases in the initial period. All in all, from these channels, agents increase private consumption, decrease saving (investment), and work more. Output increases more than government consumption does, and we can see this point more clearly in Table 4.4.

In Table 4.4, we show fiscal multipliers for output, consumption, and investment. We followed Leeper, Plante and Traum (2010) and Mountfold and Uhlig (2009) and report present value multipliers, which take into account the overall dynamics associated with fiscal shocks and properly discount future macroeconomic effect;

\[
PV \text{ Multiplier}(k) = \frac{E_t \sum_{j=0}^{k} (\prod_{i=0}^{j} R_{t+i}^{-1}) \Delta Y_{t+j}}{E_t \sum_{j=0}^{k} (\prod_{i=0}^{j} R_{t+i}^{-1}) \Delta CG_{t+j}}
\]

Note that the present-value multiplier at \( k = 1 \) is equal to impact multiplier at period 1. The impact output multiplier to CG shock amounts to 1.81.

<table>
<thead>
<tr>
<th>Variable</th>
<th>1 quarter</th>
<th>4 quarters</th>
<th>8 quarters</th>
<th>12 quarters</th>
<th>20 quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta Y / \Delta CG )</td>
<td>1.81</td>
<td>1.41</td>
<td>1.28</td>
<td>1.28</td>
<td>1.34</td>
</tr>
<tr>
<td>( \Delta CP / \Delta CG )</td>
<td>1.04</td>
<td>0.91</td>
<td>0.88</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>( \Delta IP / \Delta CG )</td>
<td>-0.22</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.58</td>
<td>-0.46</td>
</tr>
<tr>
<td>( \Delta Y / \Delta IG )</td>
<td>0.75</td>
<td>0.71</td>
<td>0.72</td>
<td>0.70</td>
<td>0.86</td>
</tr>
<tr>
<td>( \Delta CP / \Delta IG )</td>
<td>-0.17</td>
<td>-0.19</td>
<td>-0.21</td>
<td>-0.24</td>
<td>-0.29</td>
</tr>
<tr>
<td>( \Delta IP / \Delta IG )</td>
<td>-0.07</td>
<td>-0.16</td>
<td>-0.21</td>
<td>-0.23</td>
<td>-0.22</td>
</tr>
<tr>
<td>( \Delta Y / \Delta CG )</td>
<td>1.67</td>
<td>1.33</td>
<td>1.21</td>
<td>1.21</td>
<td>1.27</td>
</tr>
<tr>
<td>( \Delta CP / \Delta CG )</td>
<td>0.88</td>
<td>0.78</td>
<td>0.77</td>
<td>0.74</td>
<td>0.70</td>
</tr>
<tr>
<td>( \Delta IP / \Delta CG )</td>
<td>-0.20</td>
<td>-0.45</td>
<td>-0.56</td>
<td>-0.53</td>
<td>-0.43</td>
</tr>
<tr>
<td>( \Delta Y / \Delta IG )</td>
<td>0.76</td>
<td>0.62</td>
<td>0.68</td>
<td>0.84</td>
<td>1.23</td>
</tr>
<tr>
<td>( \Delta CP / \Delta IG )</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.11</td>
<td>0.26</td>
</tr>
<tr>
<td>( \Delta IP / \Delta IG )</td>
<td>-0.18</td>
<td>-0.35</td>
<td>-0.36</td>
<td>-0.27</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
Figure 4: Impulse responses of estimated DSGE model (Baseline Model) (government investment shock)

IG: Government Investment, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate
Blue Solid line: Impulse responses, Green and Red dotted Lines: Upper and Lower bounds of 90% HPD interval.
Figure 5: Impulse responses of estimated DSGE model (Baseline Model, Public capital in PF Model) (government investment shock)


Figures 4 and 5 illustrate impulse responses to government investment shock. Figure 4 illustrates impulse response of the Baseline model (model without public capital in production function), and Figure 5 compares the impulse response of Baseline model with that of the model with productive public capital. These figures capture important channels, crowding-out effects, wealth effects, and the effects from productive public capital to change the marginal productivity of private inputs.

For Figures 4, the first two channels are working as in the CG shock: higher government investment absorbs existing resources so that there are fewer goods available for the private sector to save (invest) or consume. As goods today become more valuable and not all intermediate goods firm can adjust its price, the real marginal cost of intermediate goods firms, rental cost of private capital, and real wages increases; higher government investment financed by lump-sum tax generates a negative wealth effect, encouraging agents to work harder, so this leads to increase in output but decrease marginal productivity of labor (real wages is affected a decreasing effect). So Figure 4 shows that agents decrease consumption, decrease saving (investment), and work more. As to real wages, the impulse response is in negative region because the first effect of real marginal cost increase was not enough to dominate the second effect of the decreasing marginal productivity of labor, by our parameters estimated. Output increase is less than government investment, and from Table 4.4, output multiplier to IG shock is 0.75 and does not exceed 1 in the initial period.

For Figure 5, however, when government investment is productive, there is another wealth effect from the opposite direction and the productive public capital effects to change the marginal productivity of private inputs. As for wealth effects, a higher stock of productive public capital acts like a total factor productivity increase to create the expectation that more goods will be available in the future, and this discourages current savings and encourages current consumption. We can see from Figure 5 that private investment decreases more and consumption decreases less in initial periods than in the baseline model.

In addition, as for the model with productive public capital, the marginal product of labor and private capital increases at longer horizons as a gradually rising stock of public capital, which results in higher wages and a return to private capital. This brings incentives to work and invest, which induces higher output in later periods. As to real wages, among these three effects, the third effect of increase in marginal productivity of labor by productive public capital dominates even in the initial periods and impulse response of real wages come up to the positive region all the time. From Table 4.4, we can see output multiplier to an IG shock in the initial period is 0.76 -not much different from baseline case- but after the reflection of the later period until 20 quarters, an output multiplier of the model with productive public capital is as high as 1.23, while the output multiplier in the baseline model stays at 0.86.

Finally, we would like to show the comparison of impulse responses of empirical models and estimated DSGE models.

In Figure 6, we showed the impulse response to government consumption shocks of the estimated DSGE model (light blue solid line), the FAVAR model
Figure 6: Impulse response comparison of empirical model and estimated DSGE model (CG shock)

In the figure, CG: Government Consumption, CP: Private Consumption, Y: output, N: Labor hours, IP: Private Investment, w: Real wage, i: Nominal Interest rate, Pi: Inflation.

Model (Light Blue Solid line): Impulse response of Baseline Model, FAVAR (Red dotted line): Impulse response of FAVAR, VAR (Blue dotted line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.
(red dotted line), the VAR model (dark blue dotted line), and the confidence intervals of the VAR model (green and purple dotted line). Overall, Figure 6 shows the estimated model succeeds in replicating impulse response obtained in VAR and FAVAR. In particular, it generates remarkably well-matched movement of private consumption, output, and labor hours. In these cases, the model-based response is similar to the ones of empirical models both in terms of magnitude and persistence. I should also mention that this model showed better movement in real wages, while Bouakez and Rebei (2007) pointed out that this aspect was deficient in their model.

In Figure 7, we showed the impulse response to government investment shocks of empirical models and two estimated DSGE models; one is the baseline model (Model 1, light blue solid line) and the other is the model with productive public capital in production function (Model 2, orange solid line). Overall, Figure 7 shows the estimated baseline model (the model without productive public capital in production function) seems better at replicating impulse responses of VAR and FAVAR than the Model 2, in that we cannot see a strong increase in output or consumption in later periods from empirical models. Another major discrepancy could be found in the response of consumption in both models. We addressed this point in the explanation of Figure 4 and 5; neoclassical models generally decrease private consumption (in initial periods) because government investment crowds out private consumption, and productive public capital works too slowly in later periods. All in all, we should admit that our estimated model with productive public capital has deficiencies in capturing the short-run movements of government investment shock although I can say that this NK model is better in capturing the short term movement of real wages. There is an area in need of more work for me.

5 Conclusion

The purpose of this paper was to see the effect of government spending on the Japanese economy using an empirical Factor Augmented Vector Autoregression (FAVAR) model and to setup a theoretical DSGE model explaining the impulse response of the empirical model and to estimate the key parameters of the model.

By the empirical FAVAR model using 107 time series of Japan, we showed that an increase in government investment and consumption leads to an increase in private consumption and real wages.

We then setup a New Keynesian general equilibrium model with real and nominal rigidities, including both Edgeworth complementarity/substitutability between private and government consumption and productive public capital. In particular, we extends Bouakez and Rebei (2007) model in three dimension; a New Keynesian model setup, including intertemporal investment adjustment cost and variable capital utilization as real rigidities, and introducing public

\footnote{See also Figure 12 and 13 in Appendix C.2.}
In the figure, IG: Government Investment, CP: Private Consumption, Y: output, N: Labor hours, IP: Private Investment, $w$: Real wage, $i$: Nominal Interest rate, $Pi$: Inflation.

Model 1 (Light Blue Solid line): Impulse response of Baseline Model, Model 2 (Orange Solid line): Impulse response of Model with productive public capital, FAVAR (Red dotted line): Impulse response of FAVAR, VAR (Blue dotted line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.
capital stocks as an externality to the production function of intermediate goods firms. This model succeeds in private consumption and real wages increase in response to government expenditure shocks.

Then, we estimated the key parameters of the model using Bayesian inference, and showed that private and government consumption are Edgeworth complements as Boaukez and Rebei (2007) found for U.S. data and Okubo (2003) and Iwata (2012) for Japanese data, and that public capital is productive in Japan. In addition, from the estimated model, we showed the impact output multiplier to government consumption shock amounts to 1.81, while that to government investment shock is 0.75.

Finally, I compared the impulse response of estimated DSGE model with those obtained by VAR and FAVAR model. Concerning government consumption shock, the estimated model succeeds in replicating impulse response by VAR and FAVAR model; it generates remarkably well matched movement of private consumption, output, labor hours, and real wages. On the other hand, our estimated model with productive public capital showed deferences in capturing the short-run movements of government investment shock. This is my very first step for model development and I will take up this issue in the future work.

A Data

All series are taken from IN information Center /INDB Finder PRO Database. Data period is from 1985Q3-2008Q1.

Data included in the observable factors $Y_t$ are the following.

1. Output: Real gross domestic product (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),
2. Private consumption: Real final consumption of household (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),
3. Private investment: Real gross capital formation of private sectors (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),
4. Government consumption: Real government final consumption expenditure (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),
5. Government investment: Real gross capital formation of public sectors (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),
6. Labor hours: Index of labor hour, total hours worked, all industries, 30 or more employees (2005=100): s.a.
7. Real wages: Index of Real Wages, total amount of cash earnings in all industries, 30 or more employees (2005=100), s.a.
8. Nominal interest rate: Call rate (uncollateralized overnight, end of month),
9. Inflation: GDP deflator, implicit deflator, s.a.

All data except inflation are transformed to logarithms and are one-sided HP filtered. As for inflation, we use one-sided HP filtered.
The other 98 variables can be seen in Table A.1. In the table, transformation codes mean 1: one-sided HP filtered, 2: logarithm and one-sided HP filtered. The circle for seasonal adjustment means we made seasonal adjustment using X12-ARIMA because seasonally adjusted series are not provided.

B Log linearized Model

For the sake of Bayesian estimation, which requires the model to be in the linear state-space form, we log-linearize the model around the steady states. We simply state the result here. The hat above a variable denotes log derivation from steady state: i.e. \( \hat{x} = \ln x - \ln x^* \) where \( x^* \) is steady state.

B.0.1 Equilibrium Conditions from Housing Sector

(1) Consumption Euler equation:

\[
\hat{x}_t = E_t[\lambda_{t+1} + \hat{R}_t - \pi_{t+1}]
\]

where

\[
\hat{x}_t = \frac{\beta \gamma (\varepsilon - 1)}{1 - \beta \gamma} E_t \hat{C}_{t+1} - \beta \gamma (\gamma (\varepsilon - 1) - 1) + \varepsilon - \frac{(1 - \beta \gamma)}{\nu} \hat{C}_t + \frac{\gamma (\varepsilon - 1)}{1 - \beta \gamma} \hat{C}_{t-1} - \frac{1}{\nu} \hat{C}_t
\]

and

\[
\hat{C}_t = \phi \left( \frac{\hat{C}^p}{\hat{C}_t} \right) + (1 - \phi) \left( \frac{\hat{C}^q}{\hat{C}_t} \right)
\]

(2) Labor Supply equation:

\[
\frac{N}{1 - \widehat{N}_t} = \hat{x}_t + \hat{w}_t
\]

(3) Investment Euler equation:

\[
\hat{I}_t^P = \frac{\beta}{1 + \beta} E_t \hat{I}_{t+1}^P + \frac{1}{1 + \beta} \hat{I}_{t-1}^P + \frac{\xi}{1 + \beta} (\hat{\mu}_t - \hat{\lambda}_t)
\]

(4) Asset Pricing Euler equation:

\[
\hat{\mu}_t - \hat{\lambda}_t = -(\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{(1 - \delta^P) E_t (\hat{\mu}_{t+1} - \hat{\lambda}_{t+1}) + \tau (1 + \Psi) E_t \hat{r}_{t+1}}{1 - \delta^P + \tau}
\]

(5) Private Capital Accumulation equation

\[
\hat{K}_{t+1}^P = \delta^P \hat{I}_t^P + (1 - \delta^P) \hat{K}_t^P
\]
Table A.1. The List of Informational time series for FAVAR

<table>
<thead>
<tr>
<th>Code</th>
<th>Data</th>
<th>Frequency</th>
<th>Seasonal adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Real Domestic Demand (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Real Private demand (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Real Private housing (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Real Public demand (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Real Exports of goods and services (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Real Imports of Goods and Services (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>National Income (at factor cost) : s.a., deflated by GDP deflator</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Real Compensation of Employees (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Real Consumption Expenditure of Household (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Real Durable goods consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Real Semidurable Goods Consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Real Nondurable Goods Consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Real Service Consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.</td>
<td>2 Q</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Index of Household Consumption Level – General (2005=100) : s.a.</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>15</td>
<td>Index of Household Consumption Level – Workers' household (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Consumption Expenditure – Worker’s household except Agriculture, Forestry and Fisheries household : (yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>17</td>
<td>Real Income – Worker’s household except Agriculture, Forestry and Fisheries household : (yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>18</td>
<td>Disposable Income – Worker’s household except Agriculture, Forestry and Fisheries household : (yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>19</td>
<td>Amount of Sales in whole country : Association of Department Stores (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>20</td>
<td>Sales at Large Scale Retail Store – Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>21</td>
<td>Sales at Department Store – Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>22</td>
<td>Sales at Supermarket – Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>23</td>
<td>Business Investment – Total Amount, All industry except finance and insurance industry (*1) (million yen)</td>
<td>2 Q</td>
<td>○</td>
</tr>
<tr>
<td>24</td>
<td>Business Investment – Total Amount, Manufacturing, *1) (million yen)</td>
<td>2 Q</td>
<td>○</td>
</tr>
<tr>
<td>25</td>
<td>Total Number of New Housing Construction Started – Total : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Total floor Area of New Housing Construction Started – Total (1000㎡) : s.a.</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>27</td>
<td>Total floor Area of Building Construction Started – Whole of Country, Grand Total (1000㎡)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>28</td>
<td>Cost Estimates for Building Construction Started – Whole of Country, Grand Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>29</td>
<td>Total floor Area of Building Construction Started – Whole of Country, Public Agent (1000㎡)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>30</td>
<td>Total floor Area of Building Construction Started – Whole of Country, Private Agent(1000㎡)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>31</td>
<td>Cost Estimates for Building Construction Started – Whole of Country, Public Agent (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>32</td>
<td>Cost Estimates for Building Construction Started – Whole of Country, Private Agent (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td></td>
<td>Description</td>
<td>Frequency</td>
<td>Notes</td>
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</tr>
<tr>
<td>33</td>
<td>Value of orders received for public construction (For 50 major firms) – All Public Institutions, total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>34</td>
<td>Value of orders received for public construction (For 50 major firms) – All National Government Agencies, Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>35</td>
<td>Value of orders received for public construction (For 50 major firms) – All Municipal Agencies, Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>36</td>
<td>Value of orders received from public institutions (5 million yen or more cases) – Total (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>37</td>
<td>Contracted Value of Public construction – Total : Public Works Payment Surety Statistics (million yen)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>38</td>
<td>Total Exports : Foreign Trade Statistics (million yen) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Total Imports : Foreign Trade Statistics (million yen) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Index of Inventory–shipment ratio of finished goods – Mining and Manufacturing (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Index of Producers’ Inventory – Mining and Manufacturing (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Index of Industrial Production – Mining and Manufacturing (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Index of Producer’s shipments – Mining and Manufacturing (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>Index of Capital Utilization Rate – Manufacturing (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>Index of Capital Utilization Rate – Manufacturing except Machinery (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>Total Amount of Electric Power Sold by 9 Major Power Companies (1000KWH)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>47</td>
<td>Current Profits – All industry except finance and insurance industry, Total Amount ((\times 1)) (million yen)</td>
<td>2 Q</td>
<td>○</td>
</tr>
<tr>
<td>48</td>
<td>Number of Business Failure</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>49</td>
<td>Labor Force (10,000 persons): s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>Number of employees (10,000 persons): s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>Number of Employer (10,000 persons): s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>Employment Index of Regular Workers: (All industries, 30 Employees or more) (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>Index of Labor Hours : Extra Working Hours, all industries, 30 Employees or more (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>Real Wage Index : Contractual Cash Earnings in All Industries, 30 Employees or more (2005=100) : s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>Unemployed (10,000 persons): s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Unemployment Rate(%) : s.a.</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>Jobs-to-Applicants Ratio : s.a.</td>
<td>1 G</td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>New Job Opening-to-Applicants Ratio : s.a.</td>
<td>1 G</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>Number of Job Offers to Job Seekers : s.a.</td>
<td>2 G</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Number of New Openings : s.a.</td>
<td>2 G</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>Domestic Wholesale Price Index – All Commodities Average, yen base (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>Export Price Index – All Commodities Average, yen base (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Import Price Index – All Commodities Average, yen base (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>Corporate Services Price Index – All Services Average, yen base (2005=100)</td>
<td>2 M</td>
<td>○</td>
</tr>
<tr>
<td>65</td>
<td>Nikkei Commodity Price Index (17 items) (End of Month)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>Nikkei Commodity Price Index (42 items) (End of Month)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Description</td>
<td>Frequency</td>
<td>Notes</td>
</tr>
<tr>
<td>---</td>
<td>--------------------------------------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>67</td>
<td>CPI: General, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>CPI: General excluding Fresh Food, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>69</td>
<td>CPI: General excluding Fresh Food &amp; Energy, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>CPI: Goods, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>CPI: Cost of Public Services &amp; Gas &amp; Electricity &amp; Water Charges, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>CPI: Services, (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>Monetary Base, no adjustment for Reserve Requirement Changes, Average Outstanding (100 million yen): s.a.</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>Monetary Base, Note Issue, Average Outstanding (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>The Basic Discount Rate and Basic Loan Rate (official discount rate) (End of Month) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>76</td>
<td>Long-term Prime Lending Rate (End of Month) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>77</td>
<td>Short-term Prime Lending Rate of banks (End of Month) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>78</td>
<td>Tokyo Stock Price Index (TOPIX): First Section of the Tokyo Securities Exchange (End of Month) (1968=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>Nikkei Stock Average 225 Selected Stocks (End of Month) (yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Foreign Exchange Rate: Tokyo Interbank, Market Spot rate (17h), end of month (yen/$)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>Foreign Effective Exchange Rate: Nominal BIS method (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>Foreign Effective Exchange Rate: Real, BIS method (2005=100)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>Yield of Interest-Bearing Government Bond (10 years) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>Yield of Local Government Bond (10 years) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>Yield of Government Guaranteed Bond (10 years) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>86</td>
<td>Yield of Interest Bearing Bank Debentures (5 years) (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>87</td>
<td>Spread bet. Long-term Prime Lending Rate and The Basic Discount Rate and Basic Loan Rate (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>88</td>
<td>Spread bet. Yield of Interest-Bearing Government Bond (10 years) and The Basic Discount Rate and Basic Loan Rate (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>89</td>
<td>Spread bet. Yield of Interest Bearing Bank Debentures (5 years) and The Basic Discount Rate and Basic Loan Rate (%)</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>Money Stock: M2+CD (End of Month): s.a. (million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>Balance on Taxation (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>Balance on Non-Tax Revenue (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>Balance on Defense-related Expenditure (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>Balance on Public Works Spendings (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>Balance on Distribution of local allocation tax (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>Balance on Government Financial Contribution for compulsory education (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>Balance on Cost of Social Security (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>Balance on Other Spendings of General Account (*2) (100 million yen)</td>
<td>2 M</td>
<td></td>
</tr>
</tbody>
</table>

*1 Data is from 'Financial Statements Statistics of Corporations by Industry, Quarterly'  
*2 Data is from 'Receipts and Payments of treasury accounts with the private sector'  
s.a. Series are seasonally adjusted
B.0.2 Equilibrium Conditions from Firm Sector

(6) Cost minimization condition:

\[ \widehat{N}_t = -m\bar{\omega} + (1 + \Psi)\widehat{r}_t + \widehat{K}_t^p \]  \hspace{1cm} (48)

(7) Production Function:

\[ \widehat{Y}_t = \widehat{A}_t + \alpha\widehat{K}_t^p + \alpha\Psi\widehat{r}_t + (1 - \alpha)\widehat{N}_t + \mu K_t^G \]  \hspace{1cm} (49)

(8) Inflation Low of Motion:

\[ \pi_t = \beta E_t\pi_{t+1} + \frac{(1-\rho)(1-\beta\rho)}{\rho}mc_t \]  \hspace{1cm} (50)

\[ mc_t = \{(1-\alpha)\bar{\omega}_t + \alpha\widehat{r}_t - \widehat{A}_t - \mu K_t^G\} \]  \hspace{1cm} (51)

B.0.3 Miscellaneous Conditions and Market Clearing Condition

(9) Public Capital Accumulation equation:

\[ \widehat{K}_t^{G_t} = \delta^G I_t^G + (1 - \delta^G)K_t^G \]  \hspace{1cm} (52)

(10) Monetary Policy Rule:

\[ \widehat{R}_t = \rho_r \pi_{t-1} + (1 - \rho_r)\phi_{rr} \pi_{t-1} + (1 - \rho_r)\phi_{rr} \widehat{Y}_t \]  \hspace{1cm} (53)

(11) Market Clearing Conditions:

\[ \widehat{Y}_t = \frac{\bar{C}_t}{\bar{Y}} \widehat{C}_t^p + \frac{\bar{C}_t}{\bar{Y}} \widehat{C}_t^G + \frac{\bar{P}_t}{\bar{Y}} \widehat{P}_t^p + \frac{\bar{P}_t}{\bar{Y}} \widehat{P}_t^G \]  \hspace{1cm} (54)

B.0.4 Persistent Shocks and Forecast Errors

\[ \widehat{A}_t = \rho_A \widehat{A}_{t-1} + \epsilon_t^A \]  \hspace{1cm} (55)

\[ \widehat{I}_t^G = \rho_{IG} \widehat{I}_{t-1}^G + \epsilon_t^{IG} \]  \hspace{1cm} (56)

\[ \widehat{C}_t^G = \rho_{CG} \widehat{C}_{t-1}^G + \epsilon_t^{CG} \]  \hspace{1cm} (57)

\[ \zeta_t^C = \widehat{C}_{t} - E_{t-1} \widehat{C}_t \]
\[ \zeta_t^\pi = \widehat{\pi}_t - E_{t-1} \widehat{\pi}_t \]
\[ \zeta_t^A = \widehat{A}_t - E_{t-1} \widehat{A}_t \]
\[ \zeta_t^\mu = \widehat{\mu}_t - E_{t-1} \widehat{\mu}_t \]
\[ \zeta_t^{I^p} = \widehat{I}_{t}^p - E_{t-1} \widehat{I}_t^{p} \]
\[ \zeta_t^{\pi} = \widehat{\pi}_t - E_{t-1} \widehat{\pi}_t \]
B.0.5 System of Log-linearized Model

From equations (2.01) through (2.023), the system of the log-linearized model are integrated as

\[
\begin{align*}
\mathbf{s}_t &= \mathbf{s}_{t-1} + \mathbf{Y}\varepsilon_t + \Pi\zeta_t \\
\end{align*}
\]

where \(\mathbf{s}_t\) is a vector of endogenous variables: \(\mathbf{s}_t = [\bar{Y}_t, \bar{\pi}_t, \bar{C}_t, \bar{C}_t^p, \bar{I}_t, \bar{\lambda}_t, \bar{\mu}_t, \bar{\omega}_t, \bar{\epsilon}_t, \bar{\omega}_t, \bar{K}_{t+1}, \bar{K}_{t+1}^G, \bar{R}_t, \bar{A}_t, \bar{I}_t^G, \bar{C}_t^G, \bar{E}_t, \bar{\lambda}_{t+1}, \bar{E}_t\bar{\mu}_{t+1}, \bar{E}_t\bar{I}_t]
\]

, \(\bar{E}_t\bar{C}_{t+1}, \bar{E}_t\bar{r}_{t+1}\)'', and \(\varepsilon_t\) is a vector of endogenous shocks: \(\varepsilon_t = [\varepsilon_t^A, \varepsilon_t^{CG}, \varepsilon_t^G]'\). \(\zeta_t\) is a vector of forecast errors: \(\zeta_t = [\xi^C_t, \xi^C_t^G, \xi^p_t, \xi^p_t^G]'\). \(\Gamma_0, \Gamma_1, \mathbf{Y}, \) and \(\Pi\) are the matrices of parameters.

C Model Implications and Calibrations

In this Appendix, we would like to show some brief implications of our model using calibration results, and compare ours with Bouakez and Rebei (2007) and Marzo and Marattin (2010).

C.1 Bouakez and Rebei (2007) model

First, we would like to show a log-linearized Bouakez and Rebei (2007) model.

1) Consumption Euler equation:

\[
\begin{align*}
\hat{\lambda}_t &= \beta\gamma(\varepsilon - 1) - \frac{1 - \beta\gamma}{1 - \beta\gamma} - \frac{1 - \beta\gamma}{\nu} \hat{C}_t + \gamma(-1 - \beta\gamma) \hat{C}_{t-1} - \frac{1}{\nu} \hat{C}_t^p \\
\end{align*}
\]

and

\[
\hat{C}_t = \phi \left( \frac{\hat{C}_t^p}{C} \right)^{\nu - 1} + (1 - \phi) \left( \frac{\hat{C}_t^G}{C} \right)^{\nu - 1} \hat{C}_t^G
\]

2) Labor Supply equation:

\[
\begin{align*}
\frac{N}{\bar{N}} \bar{N}_t &= \hat{\lambda}_t + \bar{w}_t \\
\end{align*}
\]

3) Asset Pricing Euler equation:

\[
\begin{align*}
\hat{\lambda}_t &= \frac{\hat{\lambda}_{t+1} + \beta \bar{\pi}_t}{1 + \beta \hat{\pi}_t} \hat{K}_{t+1}^P - \frac{1 + \beta \hat{\pi}_t}{\xi} \hat{K}_{t+1}^P + \frac{\hat{\lambda}_{t+1}}{\xi} \hat{K}_{t+1}^P + \frac{\beta \hat{\pi}_t}{\xi} \hat{I}_{t+1}^P \\
\end{align*}
\]

4) Private Capital Accumulation equation:

\[
\begin{align*}
\hat{K}_{t+1}^P &= \delta^P \hat{K}_t^P + (1 - \delta^P) \hat{K}_t^P \\
\end{align*}
\]
(5) Cost minimization condition:

\[
\tilde{N}_t = -\tilde{w}_t + \tilde{r}_t + \tilde{K}_t^P \\
\tilde{r}_t = \tilde{Y}_t - \tilde{K}_t^P
\]  

(6) Production Function:

\[
\tilde{Y}_t = \tilde{A}_t + \alpha \tilde{K}_t^P + (1 - \alpha) \tilde{N}_t
\]  

(7) Market Clearing Conditions:

\[
\tilde{Y}_t = \frac{\tilde{C}_t^p}{\bar{Y}} \tilde{C}_t^p + \frac{\tilde{C}_t^G}{\bar{Y}} \tilde{C}_t^G + \frac{\tilde{P}_t}{\bar{Y}} \tilde{P}_t + \frac{\tilde{I}_t^G}{\bar{Y}} \tilde{I}_t^G
\]  

Persistent Shocks:

\[
\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_t^A \\
\tilde{I}_t^G = \rho_I \tilde{G}_{t-1} + \varepsilon_t^I^G \\
\tilde{C}_t^G = \rho_{CG} \tilde{C}_{t-1}^G + \varepsilon_t^C^G
\]

First, notice that when \( \phi = 1 \) and \( \gamma = 0 \), consumption Euler equation (59)-(60) collapses to a normal Euler equation, and this model becomes a simple RBC model.

Second, we would like to show an implication regarding complementarity/substitutability between government consumption and private consumption following Boaukez and Rebei (2007). They showed from (59)-(60) the effects of changes in government consumption on the marginal utility of private consumption is given by

\[
\frac{\partial \tilde{H}_t}{\partial \tilde{C}_t^G} = (1 - \phi)(\frac{\tilde{C}_t^G}{\bar{C}})^{\frac{\varepsilon - 1}{\nu}} \{ \lambda - \varepsilon - \beta \gamma (\varepsilon - 1)(\gamma + 1) \}
\]

Now consider the model abstracting habit formation (\( \gamma = 0 \)) while effective consumption depends on government spending (\( \phi < 1 \)). Then we can see the elasticity of substitution \( \frac{\partial \tilde{H}_t}{\partial \tilde{C}_t^G} \) has the same sign as \( \frac{1}{\beta} - \varepsilon \). This means when the inverse of elasticity of substitution between private consumption and government consumption \( \frac{1}{\beta} \) is larger than \( \varepsilon \), government spending raises the marginal utility
of private consumption, which induce positive effects on private consumption while government consumption has a negative wealth effect\textsuperscript{22}.

To see this point further we made a calibration, as did Boaukez and Rebei (2007). For calibration, we used the parameter value in Table 4.1, and other parameters are in Table C.1. Figure 8 illustrates the calibration result when $\frac{1}{\nu} - \varepsilon > 0$ case, which means private consumption and government consumption are complements (in Figure 8, Comp), and $\frac{1}{\nu} - \varepsilon < 0$ case, which means private consumption and government consumption are substitutes (in Figure 8, Sub), compared with RBC case. (We made one deviation shock to government consumption.)

From Figure 8, we can see the complementary effect is strong enough to dominate the negative wealth effects under the parameter we use. On the other hand, when private consumption and government consumption are substitutes, government spending produces larger decreases in private consumption than the RBC case does. We also notice in this figure that when the complementary effects are strong, labor supply and output increase more while the decline in real wages come to be larger in that marginal productivity of labor decreases under stronger labor supply. This is the flip side of this functional form.

C.2 Setting up NK model with investment adjustment cost, role of capital utilization, and productive public capital

We would like next to show features of our model in Section 3 comparing with Boaukez and Rebei (2007) or Marzo and Marattin (2010). Our model is NK setup, (65) in Boaukez and Rebei model changed to NK Phillips curve (50), (51), and we allow government to issue bond (41), and need a monetary policy

\textsuperscript{22}In the static model, we use $\nu = 0.5$, $\varepsilon = 1$ (so $\frac{1}{\nu} - \varepsilon = 1 > 0$), so we saw $\tilde{C} > 0$ to government spending.
Figure 8: Caribrated Impulse responses of Bouakez and Rebei (2007) model

In the figure, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital.

RBC: RBC case, Comp: Model parameter with Private consumption and Government consumption are complements ($\nu=0.4$), Sub: Model parameter with Private consumption and Government consumption are substitutes ($\nu=0.8$).
Figure 9: Comparison of selected impulse responses of RBC models and NK models (CG shock)-1

In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment.


Rule (53) to set a nominal interest rate. We changed from capital adjustment cost setting (62) in the Boaukez and Rebei model to investment adjustment cost setting (45) and (46), and we added variable capital utilization, so $\Psi$ is included in (46), (48), and (49). To see the difference between these, we made some calibrations. We used a parameter of $\nu = 0.4$, $\gamma = 0.6$, and for the NK models we used price elasticity of demand $\eta = 6$, Calvo-price no price revision probability $p = 0.7$ and monetary policy parameters $\rho_r = 0.8$, $\rho_z = 1.5$, $\rho_Y = 0.05$. For a model with variable capital utilization, we used $\Psi = 2$. Other than those parameters, we used the same parameters as we used for Figure 8.

Figure 9 illustrates a comparison between the selected impulse response to government consumption shock of Boaukez and Rebei model (in FigureC.2, 1. B&R) and Marzo and Marattin (2010)'s model (in Figure 9, 3. MM), which extends Bouakez and Rebei (2007) model to a New Keynesian setup (with capital adjustment cost). In the same figure, we also compared the Bouakez and Rebei model, which changed with the investment adjustment cost setting (2. Inv).
First, if we compare the model of Boaukez and Rebei (2007) with Marattin and Marzo (2010) (1. B&R with 3. MM), we notice real wages increase for short by 3 MM. Under the NK setup, when there are the excess demands for goods, not all firms can increase price with desired markup, so real marginal costs increase while under the RBC setup, price immediately adjust to clear the goods market. Due to the increase in real marginal cost, real wages and the rental rate of private capital increase. At the same time, from the negative wealth effect, households work harder, which leads to increases in hours worked and output. We can see hours, output, and the rental rate of private capital are larger in 3 MM in Figure 9.

Second, we notice that IP movement is similar in 1 and 3, and these drops are more severe than the investment adjustment setting (red line, 2. Inv). Under the capital adjustment cost setting, it is costly to change the stock of capital while under the investment adjustment setting it is costly to change the flow of investment, so investment adjustment setting generates investment inertia and limits the crowding-out effect on investment, allowing employment and output to expand relatively more than under a capital adjustment cost setting.

Next, in Figure 11, we compare RBC setup & investment adjustment cost (2. Inv) with NK setup & investment adjustment cost (4. NK+Inv) and NK setup & investment adjustment cost & variable capital utilization (baseline). Here we firstly see the CG shock.

23See Figure 10 for more detail. Under our calibrated parameters, elasticity of inflation with respect to real marginal cost (=inverse of markup) in the NK Philips curve \( \frac{1}{(1-\rho)(1-\beta\rho)} \) will be 0.13, while under RBC setting, it is \( \infty \). Under RBC, AD shift brings just a price surge (movement from point A to point B), while under the NK Philips curve, price cannot move enough to take the desired markup and real marginal costs increase (movement from point A to point C).
Figure 11: Comparison of selected impulse responses of RBC models and NK models (CG shock)-2

In the figure, Y: output, N: Labor hours, \(w\): Real wage, \(r\): Rental rate of private capital, IP: Private Investment, Pi: Inflation rate.

2. Inv: Changed Benmaken and Rebei (2007) model to investment adjustment cost setting (RBC setup), 4. NK+Inv: Changed 2. Inv model with NK setup, Baseline: 4. NK+Inv model with capital utilization.
Comparing 2 with 4, we can say the same thing as 1 and 3 in that when there are excess demands for goods, not all firms can increase price with desired markup, so real marginal cost increases. Due to the increase in real marginal cost, real wages and the rental rate of private capital increase. At the same time, from the negative wealth effect, households work harder, which leads to increases in output. And increased work hours and output bring more inertial IP movement in 4 than in 2. On the other hand, we can see a sharp increase in the rental rate of capital and a steeper movement in price under 4.

Finally, if we compare 4 with the baseline, we notice variable capital utilization dampens the deficiencies in 4, the sharp rise in the rental rate of capital, with increasing the elasticity of the supply of capital services, and this in turn dampens the increase in real marginal cost and prices (see equation (50), (51)).

Then, in Figure 12, we would like to compare the same RBC setup & investment adjustment cost (2. Inv) with NK setup & investment adjustment cost (4. NK+Inv) of IG shock. This is also the same mechanism as I wrote in the explanation in Figure 4; there are crowding-out effect and negative wealth effect. I should mention here that as opposed to Figure 4, the impulse responses of real wages (of 4 NK+inv) in this Figure 12 go upward. This is because the
Figure 13: Comparison of selected impulse responses of NK models (IG shock)-2

In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment, CP: Private consumption.

persistance parameter of government investment shock $\rho_{IG}$ is smaller (0.6) in this calibrated figure, so the negative wealth effects (increase of labor supply) and the decrease of marginal productivity of labor are weaker than those of the estimated model. In this figure, the real marginal cost increase dominates the decrease of marginal productivity of labor.

In Figure 13, we would like to compare the NK setup & investment adjustment cost (4. NK+Inv), NK setup & investment adjustment cost & variable capital utilization (baseline), and NK setup & investment adjustment cost & variable capital utilization & productive public capital (Productive Capital); the last one is the model set up in Chapter 3. I do not run over the explanation as I wrote in Figure 5, but we can see the positive wealth effects caused by productive public capital produces the positive real wage movement especially in the later periods.

References


